



1. A controlled system described by the differential equation: $y''(t) + 3y'(t) + 2y(t) = e^{-3t}$
 - [a] Find the solution of that system if $y(t)$ and $x(t)$ are the system output and input respectively and the initial condition are: $y'(0) = y(0) = 0$.
 - [b] Find the final value of $y(t)$.
2. Find the loop equations for the circuit shown in Fig. (1). Then draw the block diagram using these loop equations.
3. Reduce the system shown in Fig. (2) to a single transfer function.

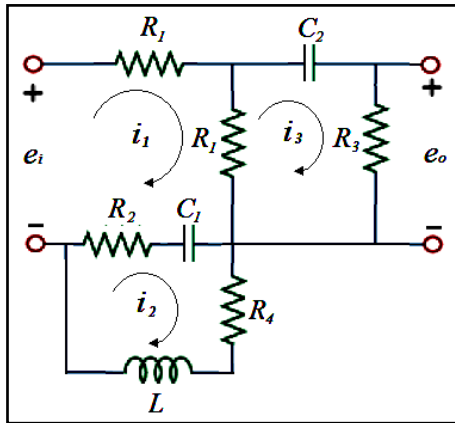


Fig. (1)

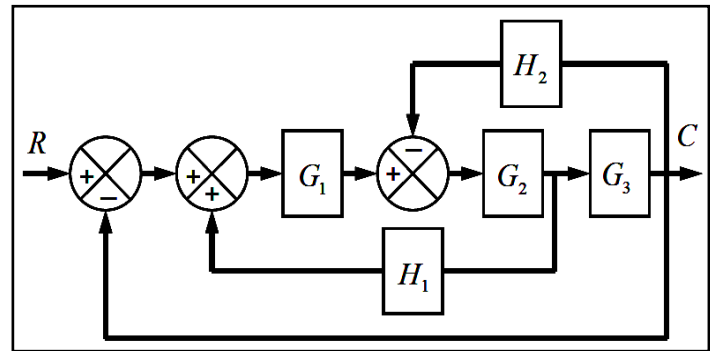


Fig. (2)



Answer

1. [a] $y''(t) + 3y'(t) + 2y(t) = e^{-3t}$

$$[S^2Y(s) - Sy(0) - y'(0)] + 3[SY(s) - y(0)] + 2Y(s) = \frac{1}{S+3} \dots\dots\dots \text{if } y'(0) = y(0) = 0$$

$$\therefore S^2Y(s) + 3SY(s) + 2Y(s) = \frac{1}{S+3} \rightarrow Y(s) = \frac{1}{(S^2 + 3S + 2)(S+3)} = \frac{1}{(S+1)(S+2)(S+3)}$$

$$\therefore Y(s) = \frac{1}{(S+1)(S+2)(S+3)} = \frac{A}{S+1} + \frac{B}{S+2} + \frac{C}{S+3}$$

First : multiply both sides by (S+1) and lets S go to -1 $\rightarrow \therefore A = \frac{1}{2}$

Second : multiply both sides by (S+2) and lets S go to -2 $\rightarrow \therefore B = -1$

Third : multiply both sides by (S+3) and lets S go to -3 $\rightarrow \therefore C = \frac{1}{2}$

$$\therefore Y(s) = \frac{1}{(S+1)(S+2)(S+3)} = \frac{A}{S+1} + \frac{B}{S+2} + \frac{C}{S+3} = \frac{\frac{1}{2}}{S+1} + \frac{-1}{S+2} + \frac{\frac{1}{2}}{S+3}$$

$$\therefore y(t) = \frac{1}{2}e^{-t} - e^{-2t} + \frac{1}{2}e^{-3t}$$

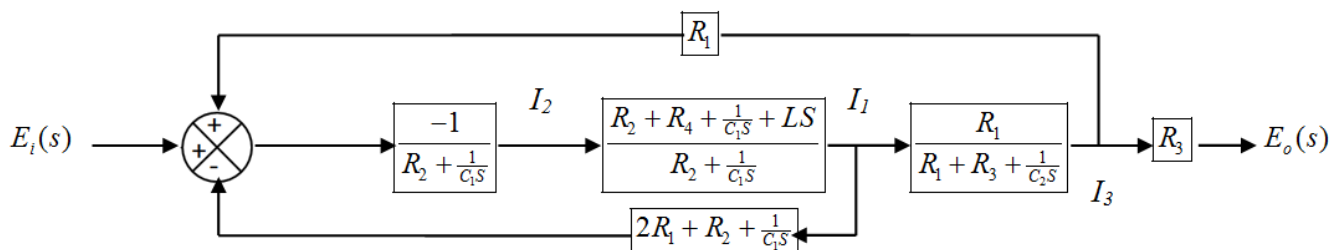
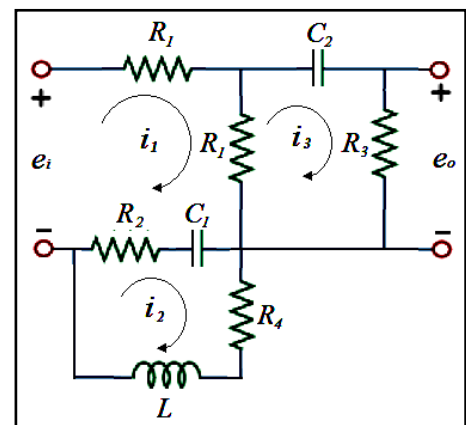
[b] $\lim_{t \rightarrow \infty} y(t) = \lim_{S \rightarrow 0} SY(s) = \lim_{S \rightarrow 0} \frac{S}{(S+1)(S+2)(S+3)} = 0$

2. $E_i(s) = I_1(s)[2R_1 + R_2 + \frac{1}{C_1S}] - I_2(s)[R_2 + \frac{1}{C_1S}] - I_3(s)R_1$

$$0 = I_3(s)[R_1 + R_3 + \frac{1}{C_2S}] - I_1(s)R_1$$

$$0 = I_2(s)[R_2 + R_4 + \frac{1}{C_1S} + LS] - I_1(s)[R_2 + \frac{1}{C_1S}]$$

$$E_o(s) = I_3(s)R_3$$





3.

